Some group-theoretical remarks on the Lambek-Grishin calculus

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In Moortgat (2007) a cube with the Grishin principles on the vertices is presented. The edges designate Grishin interactions. The lower plane consists of the interactions $(G1, G2, G4, G3)$, the upper plane of the interactions $(G1', G2', G4', G3')$.

The transformations on this cube are defined in figure 1.

Note that this picture is more precise than the slides (slide 66), which say 'the transformations † and ‡ connect vertices via the internal diagonals and ♮ via the diagonals of the standing faces'. I think this is not entirely correct because † and ‡ also use the diagonals of the standing faces.

The group $\{I, \bowtie, \sharp, \flat, \infty, \natural, \dagger, \ddagger\}$ under function concatenation is isomorphic to the group D_4. There are 8 possible isomorphisms between those groups:

I	e	e	e	e	e	e	e	e
$\dagger$	r	r	r	r	r^3	r^3	r^3	r^3
$\bowtie$	r^2	r^2	r^2	r^2	r^2	r^2	r^2	r^2
$\ddagger$	r^3	r^3	r^3	r^3	r	r	r	r
∞	t	rt	r^2t	r^3t	t	rt	r^2t	r^3t
$\flat$	r^3t	t	rt	r^2t	r^3t	t	rt	r^2t
$\natural$	r^2t	r^3t	t	rt	r^2t	r^3t	t	rt
$\sharp$	rt	r^2t	r^3t	t	rt	r^2t	r^3t	t

D_4 has 3 subgroups of order 4: $\{e, r, r^2, r^3\}$, $\{e, rt, r^2, r^3t\}$, $\{e, t, r^2, r^2t\}$. Those groups correspond to $\{I, \dagger, \bowtie, \ddagger\}$, $\{I, \sharp, \bowtie, \flat\}$ and $\{I, \infty, \bowtie, \natural\}$. The first group is isomorphic to $\mathbb{Z}_4$, the two other groups to the Klein group D_2. The group $\{I, \dagger, \bowtie, \ddagger\}$ consists of the interactions of the principles $\{G1', G2, G3', G4\}$, and of the principes $\{G1, G2', G3, G4'\}$. The group $\{I, \sharp, \bowtie, \flat\}$ corresponds to the upper and lower side planes, and the group $\{I, \infty, \bowtie, \natural\}$ consists of the planes at the front and the back of the cube. Those three pairs of subsets of principles are the only subsets of four elements closed under all relations defined by the operations. It is clear that there is no subgroup for the left and right planes, because those are not closed under † and ‡.

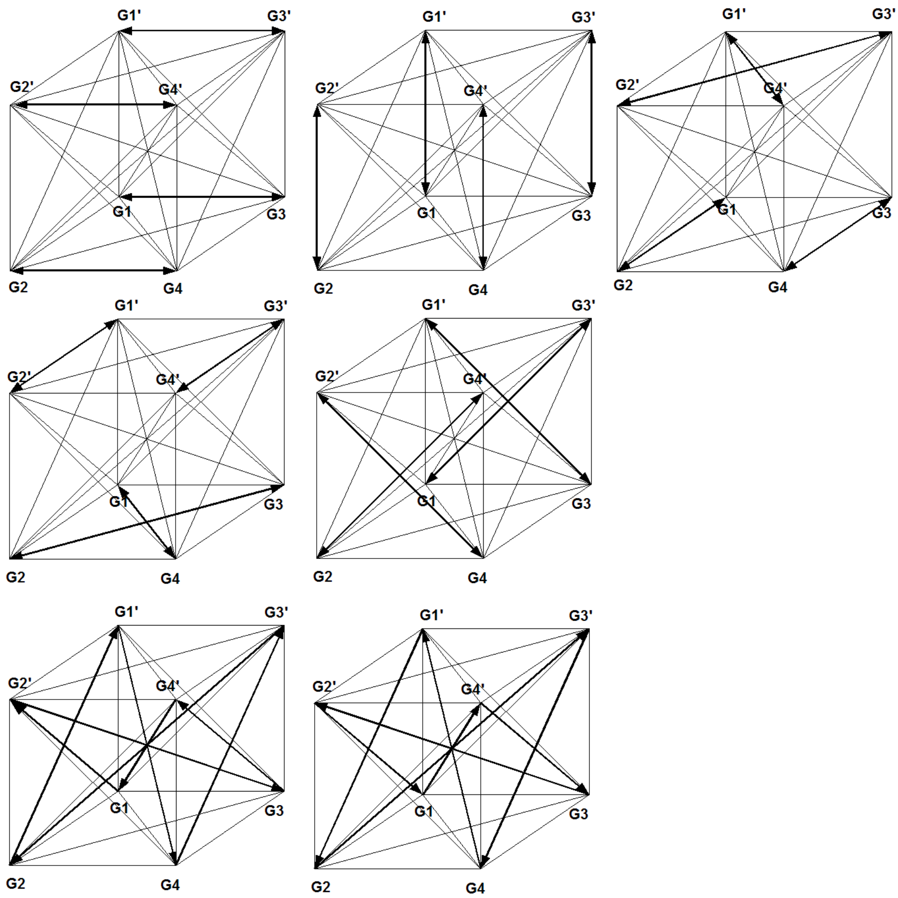

Figure 1: The Grishin interactions. First row from left to right: $\bowtie$, ∞, $\sharp$. Second row: $\flat$, $\natural$. Third row: $\dagger$, $\ddagger$.

References

[1] Michael Moortgat, Symmetric categorial grammar: syntax and semantics; Slides Logical methods in language and speech technology, Universiteit Utrecht.