A group-theoretical view of type IV Grishin principles

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In this paper I will present some group-theoretical results about a list of 24 appearances of the Grishin principles (Giorgolo, 2007). We will adept the notation of Michael Moortgat (2007), using the operators ∞, $\bowtie$, $\sharp$ and $\flat$, and the names of the principles $G1, \ldots, G4$ and $G1', \ldots, G4'$. We will call the set of 24 principles P. Furthermore we define $O = \{I, \bowtie, \sharp, \flat\}$ where I is the identity operation.

We start by making some general observations about P. First we note that for $a \in \{1, 6, 13, 16\}$ it holds that $\infty a = \bowtie a$. Therefore, the operators ∞ and $\bowtie$ cannot work both on P.

Even worse, for $a \in \{2, 5, 14, 15\}$ it holds that $\infty a = a$. Therefore, ∞ can not work at all on this group.

Grishin (1983) defines the set IV_R, which is generated by 6 inequalities (2.5 in the original paper) with $\mu = (?\oslash)$ and $\lambda = (\backslash ?)$. We can extend IV_R to P in the following way:

$$P = \{og | g \in IV_R, o \in O\}$$

We define two Grishin Principles to be equivalent when they can be derived from each other. The following set is a collection of the equivalence classes of P:

$$\{\{og | g \in IV_R\} | o \in O\}$$

Through this definition, we get the following equivalence classes: { {5,19,24,3,10,15}, {14,8,11,18,20,2}, {6,7,12,4,21,16}, {13,22,23,17,19,1} }. This is also depicted in figure 1. The correctness of this can be checked by comparing this to Giorgolo's derivations. Note that the first class contains $G3$ and $G3'$ and the second class contains $G1$ and $G1'$, but the third class contains $G4$ and $G2'$, and the forth $G2$ and $G4'$. While Gn' is defined to be ∞Gn, this means that the equivalence classes are not closed under ∞.

Alternatively, we also could define P in the following way,

$$P = \{I(\mu, \lambda) | \mu \in \{(?\oslash), (\otimes ?)\}, \lambda \in \{?/, \backslash ?\}\}$$

where $I(\mu, \lambda)$ is the list of inequivalences under 2.5 in the paper by Grishin, parametrized by λ and μ.

I did not found an elegant group which works on P, particularly because we can not use ∞. Of course it is possible to define an operator $\pm$, which cyclically maps each member in the equivalence class to the next member in this class. An alternative way of seeing this is defining $\pm$ as an operator which cyclically maps every inequality in 2.5 to the next one. If we define $\pm$ in this way, the group generated by the operators $\{\pm, \bowtie, \sharp\}$ works on P. This group is isomorphic to $\mathbb{Z}_6 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ or $\mathbb{Z}_6 \times D_2$.

Let us now look at the subsets of P. We already know that multiplying an inequality of IV_R with O gives us D_2. When we multiply the second and third inequality with O, we get $\{G_1, G_2, G_3, G_4, G'_1, G'_2, G'_3, G'_4\}$, which is equivalent to the group D_4. Also multiplying the fourth and fifth inequality with O gives us D_4. However, multiplying the first and sixth inequality does not result in D_4, because $\bowtie a = Ia$ for some of the elements of this set. We can solve this by taking the first row two times as follows: $B_1 = \{(x, false)|x \in R_1\} \cup \{(x, true)|x \in R_1\}$, where $R_1 = \{5, 14, 6, 13\}$ (the principles generated by the first inequality of IV_R), and defining $\infty^*(x, \phi) = \infty(x, \neg\phi)$. Now the group generated by ∞^*, $\bowtie$ and $\sharp$ is isomorphic to D_4 and works on B_1. We could do the same for the principles generated by the sixth inequality of IV_R.

Now we have four groups which are isomorphic to D_4. I was not able to find an intuitive relation between those four groups. This will be interesting work for the future.

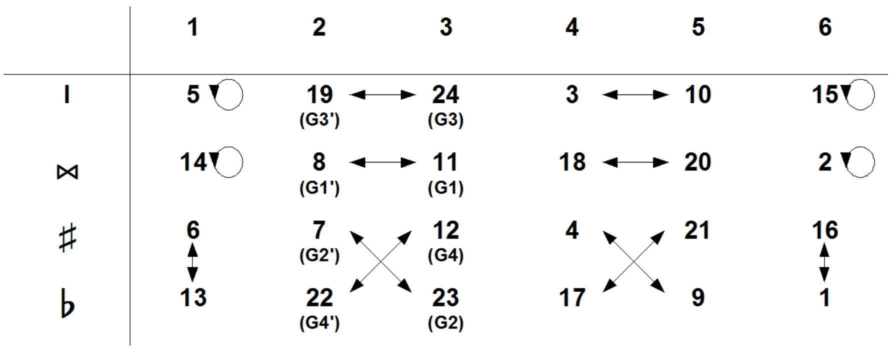

Figure 1: The 24 appearances of the Grishin principles, and the relations between them. The rows stand for equivalence classes and the columns for the set of interactions where O works on, or principles produced by one of the inequalities of IV_R. The arrows denote the result of applying ∞ to each interaction.

References

[1] The computer of Gianluca Giorgolo, 24 axioms and their proofs, 2007.

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[3] Michael Moortgat, Symmetric categorial grammar: syntax and semantics; Slides Logical methods in language and speech technology, Universiteit Utrecht, 2007.