Doubtful Deviations and Farsighted Play ⋆

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Abstract. Nash equilibrium is based on the idea that a strategy profile is stable if no player can benefit from a unilateral deviation. We observe that some locally rational deviations in a strategic form game may not be profitable anymore if one takes into account the possibility of further deviations by the other players. As a solution, we propose the concept of farsighted pre-equilibrium, which takes into account only deviations that do not lead to a decrease of the player's outcome even if some other deviations follow. While Nash equilibria are taken to include plays that are certainly rational, our pre-equilibrium is supposed to rule out plays that are *certainly irrational*. We prove that positional strategies are sufficient to define the concept, study its computational complexity, and show that pre-equilibria correspond to subgame-perfect Nash equilibria in a meta-game obtained by using the original payoff matrix as arena and the deviations as moves.

1 Introduction

The optimal strategy for an agent depends on his prediction of the other agents' behavior. For example, in security analysis, some predictions of the users' (or even the intruders') behavior can be useful when designing a particular solution. However, if the users (resp. intruders) do not behave in the predicted way, this solution might give rise to new vulnerabilities. To obtain a 'more secure' solution concept, we therefore weaken the assumptions made by agents when playing Nash equilibrium, and introduce a new solution concept based on these weaker assumptions.

Nash equilibrium (NE) defines a play to be stable when, if the players knew what the others are going to do, they would not deviate from their choices unilaterally. Conversely, if some player can beneficially deviate from strategy profile s, then the profile is assumed to describe irrational play. In this paper, we point out that some of these deviations may not be profitable anymore if one takes into account the possibility of further deviations from the opponents. As a solution, we propose the concept of *farsighted pre-equilibrium (FPE)* which takes into account only those deviations of player i that do not lead to decrease of i's outcome, even if some other deviations follow. In consequence, we argue that the notion of irrational play can be meaningfully relaxed.

Rational vs. Irrational Play We call the new concept *pre-equilibrium* because we do not imply that all FPEs are necessarily stable. Our point is rather that all strategy profiles outside FPEs are certainly *un*stable: a rational player should deviate even if he considers

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it possible that other players react to his change of strategy. Formally, FPE is strictly weaker than NE, with the following intuition: Nash equilibria correspond to play which is certainly rational, strategy profiles that are *not* pre-equilibria are certainly irrational, and the profiles in between can be rational or not, depending on the circumstances.

Farsighted Reasoning about Strategies The term "farsighted" refers to the type of reasoning about strategic choice that players are supposed to conduct according to FPE. Unlike Nash equilibrium, which assumes "myopic" reasoning (only the immediate consequences of a deviation are taken into account), farsighted pre-equilibrium looks at further consequences of assuming a particular choice. This type of strategic reasoning has been already studied for coalitional games in [2,3,4]. There have been also some attempts at farsighted stability in noncooperative games [5,6], but, as we argue in Section 5, they were based on intuitions from coalitional game theory, incompatible with the setting of noncooperative games.

Assumptions about Opponents' Play Our assumptions about the way in which players react to another player's deviation are minimal: we only assume that the reactions are locally rational. Our view of local rationality is standard for noncooperative games, i.e., it concerns an *individual* change of play that increases the payoff of the deviating player. In particular, we do not take into account scenarios where a coalition of players makes a sequence of changes that leads to a beneficial state, but leads through nodes where the payoff of some members of the coalition decreases. As we see it, such a scenario can be rational only when the coalition can commit to executing the sequence, which is not possible in noncooperative games.

Farsighted Play vs. Repeated Games An interpretation of Nash equilibrium is that a player forms an expectation about the other players' behavior based on his past experience of playing the game [1]. Then he chooses his best response strategy to maximize his immediate gain in the next instance of the game, assuming that this move will not influence future plays of the game. In other words, it is assumed that the other players do not best respond to a deviation from the expectation when the game is repeated. In contrast, in repeated games [7], it is assumed that once a player decides to deviate, the deviation will be observed by the opponents, and they will adapt to it accordingly. Then, the player would observe and adapt to their change of behavior, and so on.

In farsighted pre-equilibria, neither of these assumptions are made, as we are looking for a *weak* notion of rationality. This means that a farsighted deviation must succeed against agents that best respond farsightedly (as in the standard setting of repeated games), agents that best respond myopically, and against ones that satisfy only minimal rationality constraints (deviations must be profitable).

Structure of the Paper We begin by defining the concept of farsighted pre-equilibrium formally and discussing some examples in Section 2. We investigate how the concept behaves on the benchmark case of the n-Player Prisoner's Dilemma, provide an alternative characterization of FPE, and propose a polynomial algorithm for verifying pre-equilibria. In Section 4, we show that FPEs can be seen as subgame-perfect solutions of specific extensive form games ("deviation games"). Finally, we compare our proposal to existing work (Section 5), and conclude in Section 6.

2 Farsighted Pre-Equilibria

We begin by presenting the central notions of our proposal.

2.1 Deviation Strategies and Farsighted Stability

Let $G = (N, \Sigma_1, \ldots, \Sigma_n, out_1, \ldots, out_n)$ be a strategic game with $N = \{1, \ldots, n\}$ being a set of players, Σ_i a set of strategies of player i, and $out_i : \Sigma \to \mathbb{R}$ the payoff function for player i where $\Sigma = \Sigma_1 \times \cdots \times \Sigma_n$ is the set of strategy profiles. We use the following notation: s_i is player i's part of strategy profile s, s_{-i} is the part of $N \setminus \{i\}$, and $s \xrightarrow{i} s'$ denotes player i's deviation from strategy profile s to s' (with the obvious constraint that $s'_{-i} = s_{-i}$). Sometimes, we write $(out_1(s), \ldots, out_n(s))$ instead of s.

Definition 1. *Deviation* $s \xrightarrow{i} s'$ *is* locally rational *iff* $out_i(s') > out_i(s)$. *Function* $F_i : \Sigma^+ \to \Sigma$ *is a* deviation strategy for player i *iff for every finite sequence of profiles* $s^1, \ldots, s^k$ *we have that* $s^k \xrightarrow{i} F_i(s^1, \ldots, s^k)$ *is locally rational or* $F_i(s^1, \ldots, s^k) = s^k$. *A sequence of locally rational deviations* $s^1 \to \ldots \to s^k$ *is* F_i-compatible *iff* $s^n \xrightarrow{i} s^{n+1}$ *implies* $F_i(s^n) = s^{n+1}$ *for every* $1 \leq n < k$.

Locally rational deviations turn G into a graph in which the transition relation corresponds to Nash dominance in G. Deviation strategies specify how a player can (rationally) react to rational deviations done by other players.

Definition 2 (Farsighted pre-equilibrium). *Strategy profile s is a* farsighted pre-equilibrium *(*FPE*) if and only if there is no player i with a deviation strategy F_i such that: 1) $out_i(F_i(s)) > out_i(s)$, and 2) for every finite F_i-compatible sequence of locally rational deviations $F_i(s) = s^1 \to \ldots \to s^k$ we have $out_i(F_i(s^1, \ldots, s^k)) \geq out_i(s)$.*

This means that a strategy profile s is potentially *un*stable if there is a deviation strategy of some player i such that the first deviation is strictly advantageous, and however the other players react to his deviations so far, i can always recover to a profile where he is not worse off than he was originally in s.

Example 1. Consider the Prisoner's Dilemma game:

	C	D
C	$(\mathbf{7}, \mathbf{7})$	$(0, 8)$
D	$(8, 0)$	$(\mathbf{1}, \mathbf{1})$

The farsighted pre-equilibria are printed in bold font. The locally rational deviations are $(7,7) \xrightarrow{1} (8,0)$, $(0,8) \xrightarrow{1} (1,1)$, $(7,7) \xrightarrow{2} (0,8)$ and $(8,0) \xrightarrow{2} (1,1)$. This implies that $(1,1)$ is an FPE because there is no player i with a deviation strategy F_i such that $out_i(F_i(1,1)) > out_i(1,1)$. On the other hand, $(8,0)$ is not an FPE because $F_2(\ldots, (8,0)) = (1,1)$ is a valid deviation strategy. By symmetry, $(0,8)$ is neither an FPE. Finally we show that $s = (7,7)$ is an FPE. All deviation strategies F_1 for player 1 with $out_1(F_1(7,7)) > out_1(7,7)$ specify $F_1(7,7) = (8,0)$. Still, player 1 cannot recover from the F_1-compatible sequence of locally rational deviations $(7,7) \xrightarrow{1} (8,0) \xrightarrow{2} (1,1)$ which makes his payoff drop down to 1. The same holds for deviation strategies of player 2 by symmetry. Therefore, $(7,7)$ is an FPE.

Theorem 1. *Every Nash equilibrium is an FPE.*

Proof. Assume s is an NE. Then there exists no deviation $s \xrightarrow{i} s'$ to a strategy profile s' such that $out_i(s') > out_i(s)$. Therefore, there exists no player i with a deviation strategy F_i such that $out_i(F_i(s)) > out_i(s)$, so s is an FPE.

Corollary 1. *FPE is strictly weaker than Nash equilibrium.*

2.2 n-Person Prisoner's Dilemma

As we saw in Example 1, the Prisoner's Dilemma has two farsighted pre-equilibria: the NE profile where everybody defects, and the "intuitive" solution where everybody cooperates. This extends to the n-player Prisoner's Dilemma as defined in [5].

Definition 3 (n-Player Prisoner's Dilemma). *Let $N = \{1, 2, \ldots, n\}$ be the set of players. Each player has two strategies: C (cooperate) and D (defect). The payoff function of player i is defined as $out_i(s_1, \ldots, s_n) = f_i(s_i, h)$ where h is the number of players other than i who play C in s, and f_i is a function with the following properties:*

1. $f_i(D, h) > f_i(C, h)$ *for all* $h = 0, 1, \ldots, n-1$;
2. $f_i(C, n-1) > f_i(D, 0)$;
3. $f_i(C, h)$ *and* $f_i(D, h)$ *are increasing in* h.

The first requirement says that defecting is always better than cooperating, assuming the other players do not change their strategy. The second requirement specifies that the situation where everyone cooperates is better than the situation where everyone defects. The third requirement says that the payoff increases for a player when a larger number of the other players cooperate.

Theorem 2. *If G is a n-Player Prisoner's Dilemma, the strategy profiles $(C, \ldots, C)$ and $(D, \ldots, D)$ are FPEs in G.*

We leave out the proof because of lack of space.

Example 2. We look at an instance of the 3-player Prisoner's Dilemma. Player 1 selects rows, player 2 columns and player 3 matrices.

C	C	D
C	$(\mathbf{3}, \mathbf{4}, \mathbf{4})$	$(1, 5, 2)$
D	$(\mathbf{5}, \mathbf{2}, \mathbf{2})$	$(4, 3, 0)$

D	C	D
C	$(1, 2, 5)$	$(0, 3, 3)$
D	$(4, 0, 3)$	$(\mathbf{2}, \mathbf{1}, \mathbf{1})$

The unique Nash equilibrium is (D, D, D), the strategy profile where everyone defects, so this strategy profile is also an FPE. Furthermore, also the strategy profile where everyone cooperates, i.e., (C, C, C), is an FPE. Finally, (D, C, C) is an FPE, showing that also other FPEs can exist.

We can interpret these results as follows. A population where every player defects might be stable: being the first to cooperate is not necessarily advantageous, as the other players might not follow. A population where all players cooperate might also be stable if the players consider long-term consequences of damaging the opponents' payoffs: if one player starts defecting, the other players might follow. Finally, a strategy profile might also be stable if there are only a couple of defecting agents in the population, and the cooperating players all receive payoffs above some minimal "threshold of fairness" (which is usually the player's payoff in the Nash equilibrium $(D, \ldots, D)$). Hence the asymmetry: (D, C, C) is farsighted stable, but (C, C, D) and (C, D, C) are not, because they provide player 1 with an "unfair" payoff, and player 1 is better off heading for the NE. Another motivation for (D, C, C) to be stable, is that player 1 does not want to cooperate in the hope that players 2 and 3 do not change their strategy (as is assumed by NE), while players 2 and 3 do not want to defect out of fear for follow-ups of the other players (as is assumed in repeated games).

3 Characterizing and Computing Farsighted Pre-Equilibria

In general, deviation strategies determine the next strategy profile based on the full history of all preceding deviations. In this section, we show that it suffices for the definition of FPE to consider only *positional* deviation strategies, i.e. strategies that determine the next deviation only based on the current strategy profile, independently of what previously happened.

Definition 4. *A* positional deviation strategy *for player i is a strategy F_i such that $F_i(s^1, \ldots, s^k) = F_i(t^1, \ldots, t^k)$ whenever $s^k = t^k$. We will sometimes write $F_i(s^k)$ instead of $F_i(s^1, \ldots, s^k)$ for such strategies. A* positional FPE *is an FPE restricted to positional deviation strategies.*

Theorem 3. *A strategy profile $s \in \Sigma$ is an FPE iff it is a positional FPE.*

Proof. It suffices to prove that there is no player i with a deviation strategy such that conditions 1) and 2) from Definition 2 hold iff there is no player i with a *positional* deviation strategy such that these conditions hold. Every positional deviation strategy is a deviation strategy, so the 'only if' direction is trivial. We prove the 'if' direction by contraposition. Assume there exists a player i with a deviation strategy such that conditions 1) and 2) from Definition 2 hold in s. Now we define a positional deviation strategy F' as follows. For all $s' \in \Sigma$ for which there exist finite F_i-compatible sequences of locally rational deviations $F_i(s) = s^1 \to \ldots \to s^k = s'$, let $F_i(s) = t^1 \to \ldots \to t^k = s'$ be a shortest F_i-compatible sequence of locally rational deviations. Then we set $F'(s^k) = F(s^0, s^1, \ldots, s^k)$. For all other $s' \in \Sigma$, we set $F_i'(s') = s'$. The function F' is clearly positional and a deviation strategy. Because $F_i'(s)$ is defined based on the shortest sequence which is s (the only sequence of length 1), $F_i(s) = F_i'(s)$, and since we assumed that condition 1) holds for F_i, it also holds for F_i'. Finally we need to check that condition 2) holds. Assume $F_i'(s) = s^1 \to \ldots \to s^k$ is a finite F_i'-compatible sequence of locally rational deviations. Then by definition of F_i', there exists also a finite F_i-compatible sequence of locally rational deviations

$F_i(s) = t^1 \to \ldots \to t^{k-1} \to s^k$ with $F_i(t^1, \ldots, t^{k-1}, s^k) = F'_i(s^k)$. By assumption, $out_i(F_i(t^1, \ldots, t^{k-1}, s^k)) \geq out_i(s)$, so also $out_i(F'_i(s^k)) \geq out_i(s)$.

The following theorem provides an alternative characterization of farsighted play.

Theorem 4. *L is the set of FPEs iff for all $s \in L$, all $i \in N$ and all positional deviation strategies F_i with $F_i(s) \neq s$, there exists a finite F_i-compatible sequence of locally rational deviations $s \xrightarrow{i} s^1 \to \ldots \to s^k$ such that $out_i(F_i(s^1, \ldots, s^k)) < out_i(s)$.*

Proof. By Definition 2, a strategy profile s is an FPE iff there is no player i with a deviation strategy F_i such that: 1) $out_i(F_i(s)) > out_i(s)$, and 2) for every finite F_i-compatible sequence of locally rational deviations $F_i(s) = s^1 \to \ldots \to s^k$ we have $out_i(F_i(s^1, \ldots, s^k)) \geq out_i(s)$. Because F_i is a deviation strategy, condition 1) is equivalent to $F_i(s) \neq s$ by Definition 1. By using this equivalence and moving the negation inwards, we find that a strategy profile s is an FPE iff for every player i and all deviation strategies F_i such that $F_i(s) \neq s$, there exists a finite F_i-compatible sequence of locally rational deviations $s = s^1 \to \ldots \to s^k$ such that $out_i(F_i(s^1, \ldots, s^k)) < out_i(s)$. By Theorem 3, the theorem follows.

Now we will present a procedure that checks if the strategy profile s is a farsighted pre-equilibrium in game G. Procedure $dev(G, i, s)$ returns *yes* if player i has a successful deviation strategy from s in G, and *no* otherwise:

1. **forall** $j \in N$ **do** compute $\prec_j \in \Sigma \times \Sigma$ st. $t \prec_j t'$ iff $\exists t \xrightarrow{j} t'.\ out_j(t) < out_j(t')$;
2. let $\prec_{-i} := \bigcup_{j \neq i} \prec_j$ and let $\ll^*$ be the transitive closure of $\prec_{-i}$;
3. let $Good := \{t \mid out_i(t) \geq out_i(s)\}$; /profiles at least as good as s/
4. **repeat**
   $Good' := Good$;
   forall $t \in Good$ **do**
   if $\exists t' \gg^* t.\ (t' \notin Good \wedge \forall t' \xrightarrow{i} t''.\ t'' \notin Good)$ **then** remove t from $Good'$;
   until $Good' = Good$;
5. **if** $\exists t \in Good.\ s \prec_i t$ **then** return *yes* **else** return *no*.

The following is straightforward.

Theorem 5. *Strategy profile s is an FPE in G iff $dev(G, i, s) = no$ for all $i \in N$.*

Note that the procedure implements a standard greatest fixpoint for a monotonic transformer of state sets. As a consequence, we get the following.

Theorem 6. *Checking if s is a farsighted pre-equilibrium in G can be done in polynomial time with respect to the number of players and strategy profiles in G.*

4 Deviations as a Game

Deviations can be seen as moves in a "meta-game" called *deviation game* that uses the original payoff matrix as arena. Transitions in the arena (i.e., players' moves in the meta-game) are given by domination relations of the respective players. In such a setting, *deviation strategies* can be seen as strategies in the deviation game. A successful deviation strategy for player i is one that gets i a higher payoff immediately (like in the case of NE) but also guarantees that i's payoff will not drop below the original level after possible counteractions of the opponents. A node in the original game is an FPE exactly when no player has a winning strategy in the deviation game.

4.1 Deviation Games

A deviation game D is constructed from a strategic game G and a strategy profile s in G, and consists of two phases. In the first phase, each player can either start deviating from s or pass the turn to the next player. If no player deviates, all players get the "neutral" payoff 0 in D. If a player i deviates, the game proceeds to the second phase in which i tries to ensure that his deviation is successful, while all other players try to prevent it. This phase is strictly competitive: if i succeeds, he gets the payoff of 1 and all the other players get -1; if i fails, he gets -1 and the other players get 1 each..

Formally, given a strategic game G and a strategy profile s, the deviation game is an extensive form game $T(G,s) = (N, H, P, out'_1, \ldots, out'_n)$, where N is the set of players as in G, H is the set of histories in the deviation game, P is a function assigning a player to every non-terminal history, and for every $i \in N$, out'_i is a function assigning the payoff for player i to every terminal history. A history in H is a sequence of nodes of the form (i, t, j), with the intended meaning that $i \in N \cup \{-\}$ is the player whose deviation strategy is currently tested (where "$-$" means that no deviation has been made yet), $t \in \Sigma$ is the current strategy profile under consideration, and $j \in N \cup \{\bot\}$ is the player currently going to play (i.e., $P(\ldots,(i,t,j)) = j$), where $\bot$ indicates that the game has terminated. The initial state is $(-, s, 1)$. For every player j, we define out'_j as follows:

- $out'_j(\ldots,(-,t,\bot)) = 0$;
- if $out_i(t) \geq out_i(s)$, then $out'_j(\ldots,(i,t,\bot)) = 1$ when $j = i$, otherwise $out'_j(\ldots,(i,t,\bot)) = -1$;
- if $out_i(t) < out_i(s)$, then $out'_j(\ldots,(i,t,\bot)) = -1$ when $j = i$, otherwise $out'_j(\ldots,(i,t,\bot)) = 1$.

Now we recursively define the set of histories H, where $\underline{i}$ is defined as $\min(N \backslash \{i\})$.

1. $(-, s, 1) \in H$.
2. If $h = \ldots,(-,s,i) \in H$ and $i+1 \in N$ then $h,(-,s,i+1) \in H$.
3. If $h = \ldots,(-,s,i) \in H$ and $i = \max(N)$, then $h,(-,s,\bot) \in H$.
4. If $h = \ldots,(-,s,i) \in H$, $s \xrightarrow{i} s'$ is a locally rational deviation and $i' \in N \backslash \{i\}$ then $h,(i,s',\underline{i}) \in H$.
5. If $h = \ldots,(i,s',i) \in H$ and $s' \xrightarrow{i} s''$ is a locally rational deviation, then $h,(i,s'',\underline{i}) \in H$.
6. If $h = \ldots,(i,s',i) \in H$, then $h,(i,s',\bot) \in H$.
7. If $h = \ldots,(i,s',i') \in H$, $i' \in N \backslash \{i\}$ and either $s' \xrightarrow{i} s''$ is a locally rational deviation or $s' = s''$, then $h,(i,s'',i') \in H$ whenever both $h,(i,s'',i') \notin H$ and $i' = i$ implies $h,(-,s'',i') \notin H$.

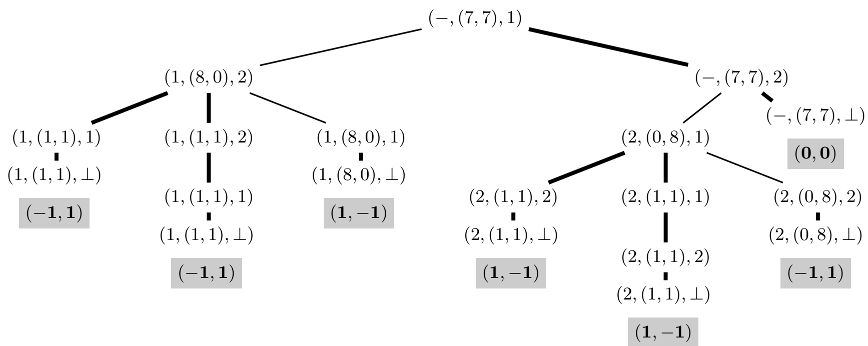

Fig. 1: Deviation game for strategy profile $(7,7)$ in Prisoner's Dilemma (Example 1)

Statement 1 specifies the initial history. Statements 2–4 say that if nobody has deviated so far, player i can embark on a deviation strategy or refrain from deviating and pass the token further. If no player deviates, the game ends. If player i initiates deviations, the strategy profile changes, and the token goes to the first opponent. Statement 5 says that the latter also applies during execution of the deviation strategy. Furthermore, 6 indicates that player i can stop the game if it is his turn (note that this can only be the case if the opponents do not want to deviate anymore). Finally, 7 states that an opponent player can make a locally rational deviation or do nothing if it is his turn, and pass the turn to another player i' (as long as the player has not had the turn in the new strategy profile before, to guarantee finite trees).

Now we can see an *opponent strategy* against player i as a set of strategies for players $N\backslash\{i\}$ such that every deviation is locally rational, and in every strategy profile, not more than one player deviates. Formally, an opponent strategy against player i is a function $F_{-i} : N\backslash\{i\} \times \Sigma^* \to \Sigma$ such that for every player $j \in N\backslash\{i\}$, $s \xrightarrow{j} F_{-i}(j,(\dots,s))$ is a locally rational deviation or $F_{-i}(j,(\dots,s)) = s$ and such that $F_{-i}(j,(\dots,s)) \neq s$ for some j implies $F_{-i}(j',(\dots,s)) = s$ for all $j' \neq j$.

Example 3. Figure 1 depicts the deviation game $T(G,s)$ where G is the Prisoner's Dilemma and s is $(7,7)$. The moves selected by the minimax algorithm are printed as thick lines. The minimax algorithm selects outcome $(0,0)$, so no player has a strategy yielding more than 0, which indicates that $(7,7)$ is an FPE.

4.2 Correspondence to FPE

Now we will prove that a strategy profile in the original game is an FPE exactly when no player has a strategy that guarantees the payoff of 1 in the deviation game. We say that a sequence of strategy profiles $s^1,\dots,s^k$ is *(F_i,F_{-i})-compatible* if for all $k' < k$ either $F_{-i}(j,(s^1,\dots,s^{k'})) = s^{k'+1}$ for some $j \in N\backslash\{i\}$ or both $F_i(s^1,\dots,s^{k'}) = s^{k'+1}$ and $F_{-i}(j,(s^1,\dots,s^{k'})) = s^{k'}$ for all $j \in N\backslash\{i\}$. Furthermore, a sequence of strategy profiles $s^1,\dots,s^k$ is *loop-free* if $s^n \neq s^{n'}$ for $1 \leq n \leq n' \leq k$.

Let G be a strategic form game, s be a strategy profile and $i \in N$ be a player. Now we say that a deviation strategy F_i is successful against an opponent strategy F_{-i}, if 1) $out_i(F_i(s)) > out_i(s)$, and 2) for every loop-free (F_i, F_{-i})-compatible sequence of strategy profiles $F_i(s) = s^1, \ldots, s^k$, it holds that $out_i(F_i(s^1, \ldots, s^k)) \geq out_i(s)$. The following lemma shows that it is indeed sufficient to look at loop-free (F_i, F_{-i})-compatible sequences.

Lemma 1. *Strategy profile s is an FPE in game G iff there does not exist a player i with a deviation strategy F_i that is successful against all opponent strategies F_{-i}.*

Proof. First we prove the 'only if' direction by contraposition. Assume there exists a player i with a deviation strategy F_i that is successful against all opponent strategies F_{-i}, i.e., 1) $out_i(F_i(s)) > out_i(s)$, and 2) for every loop-free (F_i, F_{-i})-compatible sequence $F_i(s) = s^1, \ldots, s^k$,it holds that $out_i(F_i(s^1, \ldots, s^k)) \geq out_i(s)$.

Let $F_i(s) = s^1 \to \ldots \to s^k$ be a loop-free F_i-compatible sequence of locally rational deviations. We define opponent strategy F_{-i} such that $F_{-i}(j, s^k) = s^{k+1}$ whenever $s^k \xrightarrow{j} s^{k+1}$ for $j \in N \backslash \{i\}$. Then $s^1, \ldots, s^k$ is (F_i, F_{-i})-compatible, so we have $out_i(F_i(s^1, \ldots, s^k)) \geq out_i(s)$ by assumption. Therefore, for every loop-free F_i-compatible sequence of locally rational deviations $F_i(s) = s^1 \to \ldots \to s^k$, it holds that $out_i(F_i(s^1, \ldots, s^k)) \geq out_i(s)$ $(*)$.

Now let $F_i(s) = s^1 \to \ldots \to s^k$ be a finite F_i-compatible sequence of locally rational deviations. Then we can construct a loop-free sequence $t^1, \ldots, t^k$ with $t_1 = s_1$ and $t_k = s_k$. Now $out_i(F_i(t^1, \ldots, t^k)) = out_i(F_i(s^k))$ because F_i is positional, and $out_i(F_i(t^1, \ldots, T^k)) \geq out_i(s)$ by $(*)$. Therefore, for every finite F_i-compatible sequence of locally rational deviations $F_i(s) = s^1 \to \ldots \to s^k$, it holds that $out_i(F_i(s^1, \ldots, s^k)) \geq out_i(s)$. This shows that there exists a player i with a deviation strategy F_i such that 1) $out_i(F_i(s)) > out_i(s)$, and 2) for every finite F_i-compatible sequence of locally rational deviations $F_i(s) = s^1 \to \ldots \to s^k$, it holds that $out_i(F_i(s^1, \ldots, s^k)) \geq out_i(s)$, i.e., s is not an FPE.

The 'if' direction follows from the fact that every loop-free sequence of strategy profiles is finite, and the fact that when a sequence of strategy profiles is (F_i,F_{-i})-compatible, it is also an F_i-compatible sequence of deviations.

We proceed by defining a bijection ϕ between strategy F_i in G and strategy Φ_i in $T(G, s)$ as follows.

If $F_i(s) = s$	then $\Phi_i(-, s, i) = (-, s, i+1)$	where $i + 1 \in N$;
If $F_i(s) = s$	then $\Phi_i(-, s, i) = (-, s, \bot)$	where $i = \max(N)$;
If $F_i(s) = s'$	then $\Phi_i(-, s, i) = (i, s', \underline{i})$	where $s \neq s'$;
If $F_i(s') = s'$	then $\Phi_i(i, s', i) = (i, s', \bot)$;	where $s \neq s'$;
If $F_i(s') = s''$	then $\Phi_i(i, s', i) = (i, s'', \underline{i})$	where $s \neq s' \neq s''$.

We call a set of strategies Φ_{-i} for players $N \backslash \{i\}$ *non-initially-deviating* whenever $\Phi_{i'}(-, s, i') = (-, s, i'')$ where $i \neq i'$. Then an *opponent strategy* Φ_{-i} in the deviation game is a set of non-initially-deviating strategies Φ_j for players $j \in N \backslash \{i\}$ such that in every strategy profile, not more than one player in $N \backslash \{i\}$ deviates and the other players always give the turn to the deviating player, i.e., $\Phi_j(i, s', j) = (i, s'', j')$ with $s' \neq s''$ for some $j, j' \in N \backslash \{i\}$ implies $\Phi_i(i, s', j'') = (i, s', j)$ for all $j'' \neq j$. Now we define a bijection ψ between an opponent strategy F_{-i} in T and an opponent strategy Φ_{-i} in $T(G,s)$. Let $\psi(F_{-i}) = \Phi_{-i}$, where Φ_{-i} is defined as follows:

– If $F_{-i}(i',s') \neq s'$ for some $i' \in N\backslash\{i\}$, then $\Phi_{i'}(i,s',i') = (i,s'',i')$ and $\Phi_{i''}(i,s',i'') = (i,s',i')$ for $i'' \neq i'$.
– If $F_{-i}(i',s') = s'$ for all $i' \in N\backslash\{i\}$, then $\Phi_{i'}(i,s'',i') = (i,s'',i)$.

It can easily be checked that ϕ and ψ are indeed bijections.

Let $out_i(\Phi_i,\Phi_{-i})$ be the outcome of the game for player i when player i plays strategy Φ_i and players $N\backslash\{i\}$ play strategy Φ_{-i}. When $out_i(\Phi_i,\Phi_{-i}) = u_i$ and $out_j(\Phi_i,\Phi_{-i}) = u_{-i}$ for $j \in N\backslash\{i\}$, we sometimes write $out_{i,-i}(\Phi_i,\Phi_{-i}) = (u_i,u_{-i})$.

Lemma 2. *If $i \in N$ is a player with a deviation strategy F_i and F_{-i} is an opponent strategy, then F_i is successful against F_{-i} in game G and strategy profile s if and only if $out_{i,-i}(\phi(F_i),\psi(F_{-i})) = (1,-1)$ in $T(G,s)$.*

Proof. By construction of ψ and ϕ, we have run $s = s^1 \xrightarrow{i^1} s^2 \xrightarrow{i^2} \ldots \xrightarrow{i^{k-1}} s^k$ in G iff $(-,s,1),\ldots,(-,s,i),(i,s',\underline{i}),\ldots,(i,s^k,i),(i,F_i(s^k),\underline{i}),(i,F_i(s^k),i),(i,F_i(s^k),\bot)$ is a run in $T(G,s)$. Therefore a run ends in s^k in G with $out_i(F_i(s^1,\ldots,s^k)) \geq out_i(s)$ iff a run ends in $(i,F_i(s^k),\bot)$ in $T(G,s)$ with $out_i(F_i(s^k)) \geq out_i(s)$. Therefore, F_i is successful against F_{-i} if and only if $out_i(\phi(F_i),\psi(F_{-i})) = 1$.

Theorem 7. *Strategy profile $s \in \Sigma$ is an FPE in game G if and only if all subgame-perfect Nash equilibria in $T(G,s)$ yield $(0,\ldots,0)$.*

Proof. To prove the 'only if' direction, assume strategy profile s is an FPE in game G. By Lemma 1, there does not exist a player i with a deviation strategy F_i that is successful against all opponent strategies F_{-i}. Because f and g are bijections, by Lemma 2 there does not exist a player i with a strategy Φ_i such that for every opponent strategy Φ_{-i} it holds that $out_{i,-i}(\Phi_i,\Phi_{-i}) = (1,-1)$. This means that for every player i with a strategy Φ_i, there exists an opponent strategy Φ_{-i} such that $out_{i,-i}(\Phi_i,\Phi_{-i}) \neq (1,-1)$ $(*)$. Now we prove that every subgame-perfect Nash equilibrium (SPNE) $(\Phi_1,\ldots,\Phi_n)$ in the subgame starting at $(-,s,i)$ yields $(0,\ldots,0)$ by backwards induction on $i \in (1,\ldots,n,\bot)$. The base case, where $i = \bot$, follows from the definition of out. Now assume that the claim holds for $i+1$ (where $n+1 = \bot$). To show that $\Phi_i(-,s,i) = (i,s,i+1)$, we assume that $\Phi_i(-,s,i) = (i,s',\underline{i})$ for some s' and derive a contradiction. Let Φ_{-i} be an opponent strategy. Now $out_{i,-i}(\Phi_i,\Phi_{-i})$ is either $(1,-1)$ or $(-1,1)$. If $out_{i,-i}(\Phi_i,\Phi_{-i}) = (1,-1)$, by $(*)$, there exists an opponent strategy Φ'_{-i} such that $out_{i,-i}(\Phi_i,\Phi'_{-i}) \neq (1,-1)$ and thus $out_{i,-i}(\Phi_i,\Phi'_{-i}) = (-1,1)$. Now $out_{-i}(\Phi_i,\Phi'_{-i}) > out_{-i}(\Phi_i,\Phi_{-i})$, which contradicts the assumption that (Φ_i,Φ_{-i}) is an NE. If $out_{i,-i}(\Phi_i,\Phi_{-i}) = (-1,1)$, let Φ'_i be a strategy such that $\Phi'_i(-,s,n) = (-,s,n+1)$ and Φ'_i is a SPNE strategy in the subgame starting at $(-,s,n+1)$. Then $out_i(\Phi'_i,\Phi'_{-i}) \geq 0$ for all opponent strategies Φ'_{-i} by i.h.. This implies that $out_i(\Phi'_i,\Phi_{-i}) > out_i(\Phi_i,\Phi_{-i})$, contradicting the assumption that (Φ_i,Φ_{-i}) is an NE. This implies that the assumption $\Phi_i(-,s,i) = (i,s',\underline{i})$ is false, so $\Phi_i(-,s,i) = (i,s,i+1)$ or $\Phi_i(-,s,i) = (i,s,\bot)$. By i.h., all SPNE in the subgame starting at $(-,s,i+1)$ yield $(0,\ldots,0)$. Therefore, all SPNE in $T(G,s)$ yield $(0,\ldots,0)$.

We prove the 'if' direction by contraposition. Assume strategy profile $s \in \Sigma$ is not an FPE in game G. By Lemma 1, there exists a player i with a strategy F_i that is successful against all opponent strategies F_{-i}. Because ϕ and ψ are bijections, by Lemma 2 there exists a player i with a strategy Φ_i such that for every opponent strategy Φ_{-i} it holds that $out_{i,-i}(\Phi_i, \Phi_{-i}) = (1, -1)$ (†). Now let (Φ'_i, Φ'_{-i}) be a strategy profile such that $out_i(\Phi'_i, \Phi'_{-i}) = 0$. Then there exists a strategy Φ_i such that $out_i(\Phi_i, \Phi'_{-i}) = 1$ by (†), so $out_i(\Phi_i, \Phi'_{-i}) > out_i(\Phi'_i, \Phi'_{-i})$, and therefore (Φ'_i, Φ'_{-i}) is not an SPNE. An extensive game always has an SPNE and (Φ'_i, Φ'_{-i}) is the only strategy profile yielding $(0, \ldots, 0)$, so there exist SPNEs not yielding $(0, \ldots, 0)$, which implies that not all SPNEs yield $(0, \ldots, 0)$.

Note that Theorem 7 provides an alternative way of checking pre-equilibria: s is an FPE in G iff the minimaxing algorithm [1] on $T(G, s)$ returns 0 for every player. However, the deviation game for G can be exponentially larger than G itself, so the algorithm proposed in Section 3 is more efficient.

5 Comparing Farsighted Solution Concepts

There has been a number of solution concepts with similar agenda to FPE. In this section, we discuss how they compare to our new proposal.

5.1 Related Work

The discussion on myopic versus farsighted play dates back to the *von Neumann-Morgenstern stable set* (VNM) in coalitional games [2], and Harsanyi's *indirect dominance* of coalition structures, leading to the *strictly stable set* (SSS) [3]. More recent proposals are the *noncooperative farsighted stable set* (NFSS) [6] and the *largest consistent set* (LCS) [4]. Other similar solution concepts include [8,5,9]. Also Halpern and Rong's *cooperative equilibrium* [10] can be seen as a farsighted solution concept.

Definitions In order to define the concepts, we introduce three different dominance relations between strategy profiles. *Direct dominance* of x over y means that player i can increase his own payoff by deviating from strategy profile x to strategy profile y. *Indirect dominance* of x over y says that a coalition of players can deviate from strategy profile x to strategy profile y, possibly via a number of intermediate strategy profiles, such that every coalition member's final payoff is better than his payoff before his move. Finally, *indirect dominance in Harsanyi's sense* is indirect dominance with the additional requirement that each individual deviation is locally rational. Formally:

– We say that y directly dominates x through player i ($x \prec_i y$) if there is a locally rational deviation $x \xrightarrow{i} y$. We also write $x \prec y$ if $x \prec_i y$ for some $i \in N$.
– We say that y indirectly dominates x ($x \ll y$) if there exists a sequence of (not necessarily locally rational) deviations $x = x^0 \xrightarrow{i_1} x^1 \ldots \xrightarrow{i_p} x^p = y$ such that $out_{i_r}(x^{r-1}) < out_{i_r}(y)$ for all $r = 1, 2, \ldots, p$.
– We say that x indirectly dominates y in Harsanyi's sense ($x \ll_{\mathrm{H}} y$) if there exists a sequence of locally rational deviations $x = x^0 \xrightarrow{i_1} x^1 \ldots \xrightarrow{i_p} x^p = y$ such that $out_{i_r}(x^{r-1}) < out_{i_r}(y)$ for all $r = 1, 2, \ldots, p$.

It can easily be seen that $x \prec y$ implies $x \ll_{\mathrm{H}} y$, and $x \ll_{\mathrm{H}} y$ implies $x \ll y$.

Example 4. In the Prisoner's Dilemma (Example 1), we have $(7,7) \prec_1 (8,0)$, $(0,8) \prec_1 (1,1)$, $(7,7) \prec_2 (0,8)$ and $(8,0) \prec_2 (1,1)$ In addition, $(7,7)$ indirectly dominates $(1,1)$ in Harsanyi's sense, i.e., $(1,1) \ll_{\mathrm{H}} (7,7)$.

With these definitions, we can introduce four main farsighted solution concepts.

- A subset K of Σ is a *von Neumann-Morgenstern stable set* (VNM) if it satisfies the following two conditions: (a) for all $x, y \in K$, neither $x \prec y$ nor $y \prec x$; (b) for all $x \in \Sigma \backslash K$, there exists $x \in K$ such that $x \prec y$ [2]. In fact, a VNM corresponds to stable extensions in the argumentation theory $(\Sigma, \prec')$, where $\prec'$ is the converse of $\prec$, in Dung's argumentation framework [11].
- If we replace in VNM the direct dominance relation $\prec$ by the indirect dominance relation $\ll$, we obtain the *noncooperative farsighted stable set* (NFSS) [6].
- Furthermore, a subset S of Σ is a *strictly stable set* (SSS) if it is a VNM such that for all $x, y \in S$, neither $x \ll_{\mathrm{H}} y$, nor $y \ll_{\mathrm{H}} x$ [3].
- Finally, a subset L of Σ is consistent in Chwe's sense if ($x \in L$ iff for all deviations $x \xrightarrow{i} y$ there exists $z \in L$ such that [$y = z$ or $y \ll z$] and $out_i(x) \geq out_i(z)$). Now the *largest consistent set* (LCS) is the union of all the consistent sets in Σ [4].

Another solution concept that has been recently proposed is *perfect cooperative equilibrium* (PCE) [10]. Like FPE, PCE aims at explaining situations where cooperation is observed in practice. A player's payoff in a PCE is at least as high as in any Nash equilibrium. However, a PCE does not always exist. Every game has a Pareto optimal maximum PCE (M-PCE), as defined below. We only give the definition for 2-player games; the definition for n-player games can be found in [10].

Given a game G, a strategy s_i for player i in G is a best response to a strategy s_{-i} for the players in $N \backslash \{i\}$ if $U_i(s_i, s_{-i})) = \sup_{s'_i \in \Sigma_i} U_i(s'_i, s_{-i})$. Let $BR_i(s_{-i})$ be the set of best responses to s_{-i}. Given a 2-player game, let BU_i denote the best payoff that player i can obtain if the other player j best responds, that is $BU_i = \sup_{s_i \in \Sigma_i, s_j \in BR(s_i)} U_i(s)$. A strategy profile is a PCE if for $i \in \{1,2\}$ we have $U_i(s) \geq BU_i$. A strategy profile is an α-PCE if $U_i(s) \geq \alpha + BU_i$ for all $i \in N$. The strategy profile s is an M-PCE if s is an α-PCE and for all $\alpha' > \alpha$, there is no α'-PCE.

Example 5. In the Prisoner's Dilemma (Example 1), there is one VNM ($\{(1,1),(7,7)\}$) and one NFSS ($\{(7,7)\}$). There is no SSS, and the LCS is $\{(1,1),(7,7)\}$.

Regarding PCE, we have $BR_1(C) = \{D\}$, $BR_1(D) = \{D\}$, $BR_2(C) = \{D\}$, and $BR_2(D) = \{D\}$. This implies that $BU_1 = D$ and $BU_2 = D$. Thus, the set of PCE outcomes is $\{(7,7),(1,1)\}$, and $(7,7)$ is the unique M-PCE (with $\alpha = 6$).

5.2 FPE vs. Other Farsighted Concepts

The main idea of all introduced farsighted solution concepts (except PCE) is very similar. One can test whether a given strategy profile is stable by checking whether a player or group of players can deviate from the strategy profile in a profitable way, given a possible follow-up from the other players. However, there are also many differences

	L	R
T	$(3,1)$	$(\mathbf{0},\mathbf{2})$
B	$(\mathbf{1},\mathbf{1})$	$(0,0)$

(a)

F	L	R
T	$(3,1,2)$	$(\mathbf{0},\mathbf{2},\mathbf{2})$
B	$(0,0,0)$	$(0,0,0)$

S	L	R
T	$(3,1,1)$	$(0,0,0)$
B	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$(0,0,0)$

(b)

	L	R
T	$(3,1)$	$(\mathbf{2},\mathbf{2})$
B	$(1,1)$	$(0,0)$

(c)

	L	C	R
T	(0,0)	$(3,1)$	$(\mathbf{0},\mathbf{2})$
B	$(\mathbf{1},\mathbf{3})$	$(\mathbf{2},\mathbf{4})$	$(0,0)$

(d)

Fig. 2: Example games (FPEs are printed in bold)

between the concepts. In this section, we will compare FPE with other farsighted solution concepts in various aspects.

Scope of Farsightedness In farsighted reasoning about strategies, players consider further consequences of their deviations, as opposed to reasoning in myopic solution concepts like Nash equilibrium. Consider for example the game in Fig. 2a. Strategy profile $(1,1)$ is not an NE because $(1,1) \xrightarrow{1} (3,1)$ is locally rational. However, this deviation is not necessarily *globally* rational, as it might trigger player 2 to follow up with the deviation $(3,1) \xrightarrow{2} (0,2)$. Unlike Nash equilibrium, which only considers (0,2) stable, the set $\{(1,1),(0,2)\}$ is considered stable in all presented farsighted solution concepts (VNM, NFSS, SSS, LCS and FPE).

The degree of farsightedness is different across the concepts. The least farsighted concept is VNM. Here, players only look at whether they can recover from a *single* deviation of the opponents, as the game in Fig. 2b illustrates. The deviation $(1,1,1) \xrightarrow{1} (3,1,1)$ is locally rational but might intuitively be wrong because deviations $(3,1,1) \xrightarrow{3} (3,1,2) \xrightarrow{2} (0,2,2)$ can spoil its effect. However, since VNM does not take sequences of deviations into account, it does not consider $(1,1,1)$ stable ($\{(0,2,2),(3,1,1)\}$ being the only stable set). The concepts NFSS, LCS and FPE have a "more farsighted" view, and consider sequences of follow-up deviations. In consequence, they all deem the profile $(1,1,1)$ stable.

Furthermore, the solution concepts evaluate follow-ups differently. In VNM and NFSS, a follow-up deviation from the opponents is always considered undesirable, even if it gives a higher payoff for the first deviating player. In LCS and FPE, beneficial follow-ups only strengthen the success of the original deviation. Consider the game in Fig. 2c. After $(1,1) \xrightarrow{1} (3,1)$, the follow-up $(3,1) \xrightarrow{2} (2,2)$ still leaves player 1 with a payoff higher that his initial one. Thus, both LCS and FPE deem $(1,1)$ unstable, which matches intuition, while VNM and NFSS consider $(1,1)$ stable.

Type of Solution Concept The concepts also yield objects of different types. LCS and FPE both return a set of strategy profiles, thus ascribing rationality to *individual profiles*. On the other hand, VNM, NFSS and SSS return a set of sets of profiles each, hence ascribing rationality to *sets* of strategy profiles. In the latter case a rational set of profiles can be understood as a set of collective decisions to whom the grand coalition of players can consistently stick. Clearly, this makes sense in coalitional games, but is less suitable for noncooperative games where the players' control over collective choice is limited.

Deviation Strategy VNM, SSS and FPE are built on a pessimistic view of the follow-up to the first deviation, as they make no assumptions about the other players' rationality. In particular, it is not assumed that opponents will help to increase the initiator's outcome, even if it is also to their advantage. In consequence, these solution concepts assume that the deviations of the initiator must always be locally rational. In contrast, NFSS assumes that a player can make deviations which are not locally rational if he hopes that other players will further increase his outcome. The game in Fig. 2a illustrates this. The set $\{(1,1),(0,2)\}$ is a VNM, SSS, LCS and collects all FPEs. On the other hand, $\{(0,2)\}$ is the only NFSS. PCE and M-PCE may also require players to deviate in a locally irrational way because they do not take into account the domination relation explicitly. For example, $(0,2)$ in Fig. 2d is neither PCE nor M-PCE, although it is a Nash equilibrium and hence no player has a locally rational deviation in it. All the other solution concepts considered here deem $(0,2)$ stable.

Expected Behavior of Opponents Different solution concepts imply different opponent models. We have already mentioned that the initiator of deviations can either be optimistic or pessimistic about the follow-up by the opponents. Another distinction is whether the deviator expects the opponents to be farsighted as well, or whether they might be regular best-response players. Consider the game in Fig. 2d. Intuitively, a farsighted player 1 would not deviate $(2,4) \xrightarrow{1} (3,1)$, because the follow-up deviation $(3,1) \xrightarrow{2} (0,2)$ can damage his payoff. Therefore player 2 can safely play $(1,3) \xrightarrow{2} (2,4)$ if he is sure that player 1 is farsighted. However, if player 1 plays best response, the deviation $(1,3) \xrightarrow{2} (2,4)$ might harm player 2, because player 1 will deviate $(2,4) \xrightarrow{1} (3,1)$ afterwards. Therefore, if player 2 has no information about the kind of behavior of player 1, it might be better to stick to strategy profile $(1,3)$. FPE is the only solution concept that captures this intuition by considering $(1,3)$, $(2,4)$ and $(0,2)$ to be (potentially) stable; the other formalisms (VNM, SSS, NFSS, LCS) all result in the stable set $\{(1,4),(0,2)\}$.

Summary The main difference between our farsighted pre-equilibrium and the other solution concepts discussed in this section lies in the perspective. It can be argued that the type of rationality defined in [2,3,4,5,6] is predominantly coalitional. This is because those proposals ascribe stability to *sets* of strategy profiles, which does not have a natural interpretation in the noncooperative setting. Moreover, some of the concepts are based on coalitional rather than individual deviations. On the other hand, the concept of cooperative equilibrium [10] is *not* based on reasoning about possible deviations. In this sense, FPE is the first truly noncooperative solution concept for farsighted play that we are aware of.

6 Conclusions

We have proposed a new solution concept that we call *farsighted pre-equilibrium*. The idea is to "broaden" Nash equilibrium in a way that does not discriminate solutions that look intuitively appealing but are ruled out by NE. Then, Nash equilibrium may be interpreted as a specification of play which is certainly rational, and strategy profiles that are *not* farsighted pre-equilibria can be considered certainly *irrational*. The area in between is the gray zone where solutions are either rational or not, depending on the detailed circumstances.

Our main motivation is predictive: we argue that a solution concept that makes too strong assumptions open up ways of possible vulnerability if the other agents do not behave in the predicted way. Nash equilibrium seems too restrictive in many games (Prisoner's Dilemma being a prime example). We show that FPE does select non-NE strategy profiles that seem sensible, like the "all cooperate" strategy profile in the standard as well as the generalized version of Prisoner's Dilemma. Moreover, we observe that FPE favors solutions with balanced distributions of payoffs, i.e., ones in which no player has significantly higher incentive to deviate than the others.

A natural way of interpreting deviations in strategy profiles is to view the deviations as moves in a "deviation game" played on the metalevel. We show that farsighted pre-equilibria in the original game correspond to subgame-perfect Nash equilibria in the meta-game. This is a strong indication that the concept that we propose is well rooted in game-theoretic tradition of reasoning about strategic choice.

Farsighted play has been investigated in multiple settings, starting from von Neumann and Morgenstern almost 70 years ago. Our proposal is (to our knowledge) the first truly noncooperative solution concept for farsighted play. In particular, it is obtained by reasoning about *individual* (meta-)strategies of *individually* rational players, rather than by reconstruction of the notion of *stable set* from coalitional game theory.

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